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Optimization of a Space Transportation System Including Design Sensitivities

V. Engler,* D. Coors,† and D. Jacob‡
Technical University of Aachen, 52062 Aachen, Germany

A new method for the performance rating of a space transportation system is presented. The method combines a commonly used quality criterion, the so-called growth factor, with a penalty function that penalizes high sensitivities with respect to inaccuracies in the employed physical computation models. The method is applied to a space transportation system with horizontal takeoff and landing, an air-breathing first stage, and a rocket-propelled second stage. Using a multimembered evolution strategy, an optimization of the lower stage propulsion system is performed with and without considering design sensitivities. The results show that very critical situations can arise if the sensitivity to inaccuracies in the physical models is neglected. Furthermore, remarkable reductions of the design sensitivity by more than an order of magnitude are achievable with only moderate losses in the total system performance.

Nomenclature

b	= span width, m
GF	= growth factor, gross mass/payload mass
g_i	= weighting factor, %
I_{sp}	= specific impulse, s
l	= total length, m
M	= Mach number
M_{sep}	= initialization Mach number for the stage separation
$M_{T \to R}$	= switching Mach number from turbojet to ramjet mode
$M_{z>1}$	= switching Mach number to overstoichiometric combustion
n	= number of models with assumed inaccuracies
S	= sensitivity measure
$(T/W_0)_R$	= thrust-to-weight ratio at the design point ($M = 7$, altitude = 29 km) in the ramjet mode
$(T/W_0)_T$	= thrust-to-weight ratio at the design point ($M = 0$, altitude = 0 km) in the turbojet mode
z	= stoichiometric ratio
Δ_i	= model inaccuracy in the submodel i, %
ΔGF_i	= growth factor increase caused by a model inaccuracy in the submodel i

Subscripts

FS	= first stage
SS	= second stage
0	= initial conditions

Superscripts

+	= weighted gradients approach
*	= global sensitivity approach

Introduction

AT present, many concepts of advanced space transportation systems are under investigation worldwide. Although these concepts have fundamental differences [e.g., single-stage-to-orbit vs two-stage-to-orbit (TSTO), horizontal takeoff vs vertical takeoff], they have at least one thing in common, i.e., the very strong interactions between aerothermodynamics, propulsion, structure, flight mechanics, and trajectory. Therefore, it is absolutely necessary to consider these interdisciplinary interactions in the design process and the corresponding evaluation methods that are used for the efficiency rating of a given concept as early as possible. This is well known and generally accepted (for example, see Ref. 1). However, the performance evaluation of a total system or a special configuration involves another serious problem that is only rarely discussed in the literature. The nominal performance, i.e., the corresponding value of a quality criterion, is normally determined by performing a mission simulation, assuming that all physical models used in the simulation are accurate. This means that the evaluated nominal performance inevitably depends on the quality and the accuracy of the physical models used. Especially in an early design phase these models are usually simple, and it is very unlikely that they describe reality exactly. Unfortunately, these highly complex advanced space transportation systems with many interacting parameters are very sensitive. A small variation, a simple inaccuracy in one submodel, e.g., in the drag evaluation, can have large effects on the total system performance. This becomes especially important if an optimization is intended because this usually tends to extreme design solutions, leading to configurations with increased sensitivity. Therefore, even small inaccuracies in one submodel can have disastrous effects on a supposedly optimized configuration. The objective of the analysis presented in this paper is to provide a method that takes into account the sensitivities to inaccuracies in the employed physical computation models. The intention is to find a configuration with considerably lower sensitivities and an only slightly lower nominal performance than the optimized configuration without including model inaccuracies.

Method

A commonly used quality criterion for the efficiency rating of a space transportation system is the so-called growth factor that is given by the ratio between the gross takeoff mass and the payload mass in orbit. The aforementioned growth factor definition reveals that it should be as small as possible to maximize the payload mass for a given gross mass. The growth factor value that is calculated by performing a mission simulation depends not only on the parameters of the space transportation system, e.g., geometry or propulsion parameters, but also inevitably on the quality and the accuracy of the physical models used. Therefore, it is very important to consider sensitivities with respect to inaccuracies in the employed physical models to access the design risks.

This need is demonstrated in principle by Fig. 1 for a one-dimensional example. The figure shows the dependence of the growth factor on the variation of one design parameter, for example,

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*Research Engineer, Institute of Aeronautics and Astronautics (ILR), Wüllnerstrasse 7.

†Senior Research Engineer, Institute of Aeronautics and Astronautics (ILR), Wüllnerstrasse 7.

‡Professor and Head, Institute of Aeronautics and Astronautics (ILR), Wüllnerstrasse 7. Member AIAA.

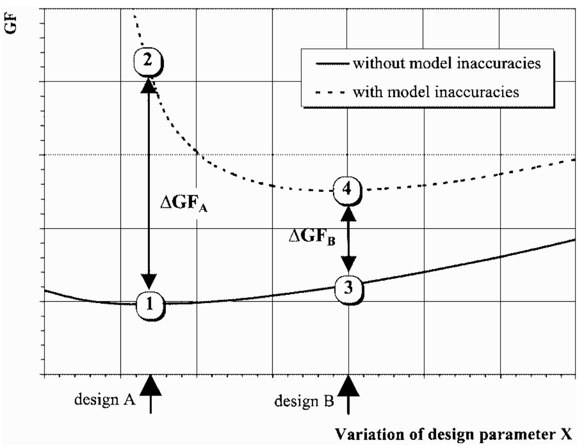

Fig. 1 Influence of model inaccuracies on optimal design selection (sketch of principle).

the installed thrust. The solid line refers to the growth factor that is obtained if all physical computation models used are supposed to be correct. This assumption means that the physical computation models used in the mission simulation describe reality exactly. The resulting growth factor is called the nominal growth factor. The growth factor values of the dashed line result if model inaccuracies in one or more computation models are assumed. This growth factor can be called a sensitivity-weighted growth factor. The difference ΔGF can be understood as a sensitivity measure. If the nominal growth factor (solid line) is the only decision criterion, design A is the best solution (point 1). If there are actual errors or inaccuracies in the employed calculation models (dotted line), the nominal growth factor is considerably worsened (point 2). If possible inaccuracies are considered right from the beginning, then design B (point 4) should be chosen because it is clearly less sensitive ($\Delta GF_B \ll \Delta GF_A$), whereas the nominal growth factor is only slightly increased (point 3).

This example shows that the nominal growth factor alone is not sufficient for the efficiency rating of a space transportation system, especially in early design phases. New or at least additional criteria are needed that include the sensitivities to model inaccuracies. In the present study the basic idea is to add a penalty function to the nominal growth factor; this means penalizing high sensitivities with respect to model inaccuracies by increasing the growth factor artificially.

Definition of Sensitivity-Based Quality Criteria

The model inaccuracy Δ_i is defined as the relative difference between reality and the computation model i. For example, a $\Delta_{\text{drag model}} = 5\%$ means that the real drag is 5% higher than the value resulting from the drag calculation models employed, keeping all design parameters, such as wing area or sweep angle, constant. The intention of the present study is to penalize sensitivities. Therefore, a worst case assumption is made. This assumption means that only model inaccuracies that lead to an increase in the growth factor are considered.

As mentioned earlier, the nominal growth factor

$$GF = GF(\Delta_i = 0, \quad i = 1, \ldots, n) \tag{1}$$

is the growth factor that results from a mission simulation assuming that there are no model inaccuracies.

A. Weighted Gradients Approach

The sensitivity-weighted growth factor GF_i results directly from one mission simulation if a model inaccuracy is considered in one computation model, i, exclusively, while all other models are assumed to be correct:

$$GF_i = GF\begin{pmatrix} \Delta_j \neq 0 & \text{for} & j = i, & \\ \Delta_j = 0 & \text{for} & j \neq i, & j = 1, \ldots, n \end{pmatrix} \tag{2}$$

The growth factor increase ΔGF_i caused by the inaccuracy Δ_i in the computation model i results from

$$\Delta GF_i = GF_i - GF \tag{3}$$

The gradient $\Delta GF_i/\Delta_i$ that is assumed to be constant for various small Δ_i is a measure for the sensitivity to inaccuracies in the computation model i. By weighting these gradients with a factor g_i and adding the obtained weighted gradients for the n different computation models, the sensitivity measure S^+ according to the method of weighted gradients can be determined:

$$S^+ = \sum_{i=1}^{n} \left| \frac{\Delta GF_i}{\Delta_i} \cdot g_i \right| \tag{4}$$

By using this sensitivity measure S^+, we can define a sensitivity-weighted growth factor GF^+ (in contrast to the nominal growth factor GF):

$$GF^+ = GF + S^+ = GF + \sum_{i=1}^{n} \left| \frac{\Delta GF_i}{\Delta_i} \cdot g_i \right| \tag{5}$$

The weighted gradients approach allows the investigation of the influence of each single model on the total sensitivity and the simulation of different inaccuracy scenarios by using different weighting factors $\boldsymbol{g} = (g_1, \ldots, g_n)$. But there are also two disadvantages. Nonlinear effects are not considered by this approach, and much computational effort is required because a total number of $n + 1$ mission simulations are necessary to determine the sensitivity-weighted growth factor GF^+.

B. Global Sensitivity Approach

The aforementioned disadvantages are avoided if inaccuracies are assumed in all n models at the same time. Then the sensitivity-weighted growth factor GF^*, according to the global sensitivity approach, results directly from only one mission simulation:

$$GF^* = GF(\Delta_i \neq 0, \quad i = 1, \ldots, n) \tag{6}$$

In this case, the sensitivity measure S^* is given by

$$S^* = GF^* - GF \tag{7}$$

Generally, the sensitivity measure S^* is higher than the sensitivity measure S^+ according to the weighted gradients approach because the global sensitivity approach includes nonlinear effects.

Altogether, there are now five quality criteria available that can be used as rating measures for a space transportation system: the nominal growth factor GF, the sensitivity-weighted growth factors GF^+ and GF^*, and last but not least the sensitivity measures S^+ and S^*. Our main interest is focused on the sensitivity-weighted growth factors because they represent the desired compromise between high nominal performance and low design risk.

Reference Configuration and Trajectory

In this paper the present method is applied to a TSTO space transportation system as proposed by the Collaborative Research Center under the program Fundamentals of Design of Aerospace Planes (SFB 253). This Collaborative Research Center was established by the German Research Association at the Technical University of Aachen in 1989. The TSTO concept used is sketched in Fig. 2. It serves as the reference concept for many basic research topics in the fields of aerodynamics, propulsion, and structures. The reference configuration for the first stage is a configuration called ELAC. It is a lifting-body configuration with an elliptical cross section. The body has a delta-shaped planform with a total length of 72.0 m, a leading-edge angle of 75 deg, and an aspect ratio of 1.1. The maximum relative thickness is 7.4% and is located at 66% of the total length. The air-breathing propulsion system is placed on the lower side of the configuration. It has to work in a very wide velocity range from $M = 0$ up to 7.0–7.5 at stage separation. Therefore, a combination of turbojet and ramjet engines is necessary. The reference

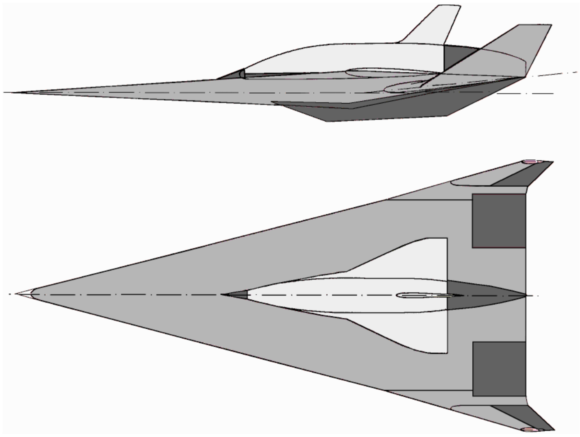

Fig. 2 Reference concept.

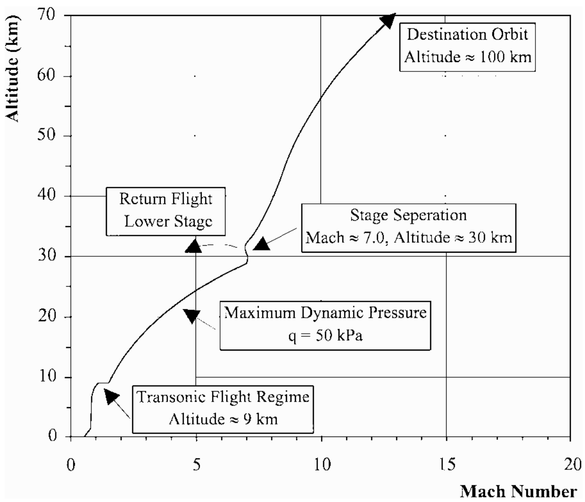

Fig. 3 Reference trajectory.

propulsion system is designed to reach an acceleration capability of at least 0.1 g during the whole flight in the turbojet mode as well as in the ramjet mode. This minimum value of 0.1 g, which is only reached in the transonic flight regime and at the stage separation, was arbitrarily chosen to limit flight time, fuel consumption, and heat loads, especially at high velocities.

The second-stage EOS is a wing–body configuration, which is similar to the HORUS configuration of the SÄNGER concept, except for the central tail fin. The second stage is partly integrated in the first stage to reduce the aerodynamic drag. It is propelled by a liquid hydrogen (LH2)/liquid oxygen (LOX) rocket engine with a specific impulse of $I_{sp} = 350$ s at sea level and 472 s in vacuum. The rocket engine provides an initial thrust-to-weight ratio of 1.25.

The initial mass of the total space transportation system is fixed to 350 tons. This is in the middle range of the worldwide proposed concepts. Because the mass of the upper stage depends on the lower-stage performance, as we will see later, it is assumed that the upper stage is scalable near its design point.

The reference trajectory is shown in Fig. 3. It is divided into an air-breathing part of the first stage and a rocket-propelled part of the second stage. It is a pure acceleration mission without any cruise flight. After a horizontal takeoff, a subsonic climb takes place up to an altitude of about 9 km. At this altitude the space transportation system accelerates up to a maximum dynamic pressure of 50 kPa, passing horizontally through the transonic velocity regime, which is characterized by maximal aerodynamic drag. Then the flight continues along a line of constant dynamic pressure (50 kPa) until Mach 7.0 at an altitude of about 30 km. A pull-up maneuver takes place to reach the stage separation conditions. After separation the first stage flies back to the base or an alternative landing site while the second stage continues its ascent to the destination orbit of 100 km.

Mission Simulation

The main tool in this investigation is the mission simulation. It is necessary to calculate the nominal growth factor and to determine the growth factor changes due to inaccuracies in the physical calculation models, respectively. The procedure used in the present study is shown in Fig. 4. The two most important tasks are the calculation of fuel and empty mass of both stages. The fuel mass of the first stage is influenced by the aerothermodynamics, the efficiency of the air-breathing propulsion system, and the trajectory. The empty mass consists of the masses of the structure, the equipment, the thermal protection system, and last but not least the mass of the air-breathing propulsion system. After calculating fuel and empty mass for a fixed initial mass, the payload mass of the first stage (equivalent to the initial mass of the second stage) can be determined easily. The payload mass of the second stage is calculated in the same manner, but in less detail.

The physical models used for the first stage, which is the focus of the present study, are briefly described next.

The aerodynamic forces are calculated as a function of angle of attack, Mach number, and altitude by using analytical models for a simplified geometry. For example, a modified lifting line theory by De Young and Harper[2] is used for subsonic Mach numbers, and a second-order elliptic cone solution described by van Dyke[3] is applied for the elliptic forebody in the supersonic and hypersonic Mach number regimes. Analytical and experimental results, carried out by colleagues at the Technical University of Aachen (for example, by Decker et al.[4] or Krause and Henze[5]), are used to verify and to complete the aerodynamic performance evaluation.

The mass analysis is performed with either analytical or statistical methods. An analytical method, based on a modified beam theory as described by Ardema,[6] is used to find the fuselage mass. The masses of the thermal protection system, fuel tanks, and other subsystems are estimated by applying statistical methods based on Ref. 7. The propulsion system mass is determined according to Steinebach.[8]

The thrust, the contribution of the propulsion system to lift and pitching moment, and the fuel mass flow as functions of angle of

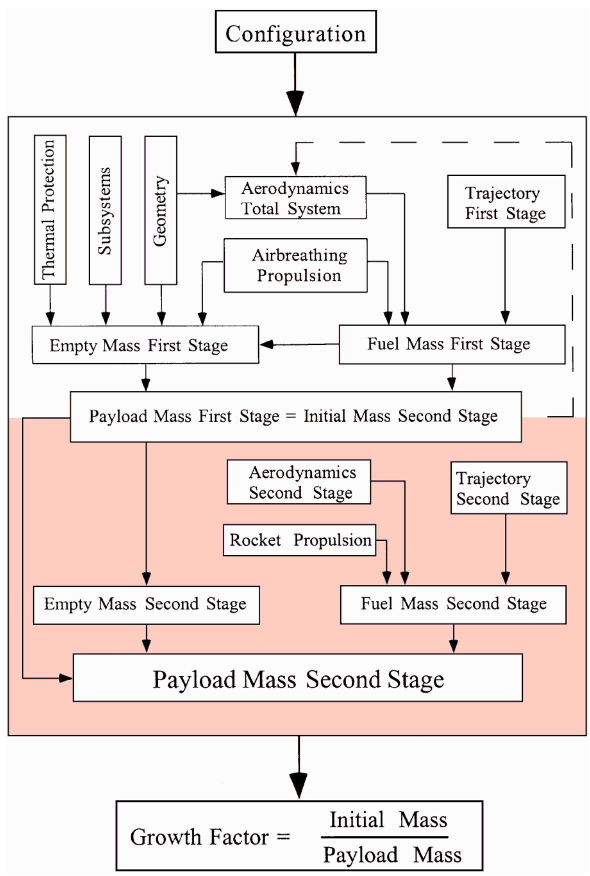

Fig. 4 Mission simulation.

attack, Mach number, and altitude are interpolated from data fields that are also based on the works of Steinebach.[8]

The state variables of the trajectory are determined by integrating the equations of motion. The total fuel mass is found by integrating the fuel flow along the trajectory. Note that the ascent of both stages takes place in the equatorial plane of a spherical, rotating Earth. The atmospheric data of the flight path are taken from Ref. 9.

Optimization Algorithm

An air-breathing space transportation system is a very complex system with many interacting parameters. These interactions require a simultaneous multiparametric optimization. Another fact is that it is impossible to give a complete analytical description of the total system, but many optimization methods require such an analytical description. These constraints make the optimization of a space transportation system a very difficult task. Therefore, a multimembered evolution algorithm was chosen as the optimization strategy. Evolution strategies imitate the natural evolution process. They are very robust concerning the convergence behavior, even for highly complicated multiparametric systems, and they do not need any information on the inner mathematical connections of the system that has to be optimized.[10]

Detailed information about evolution strategies are given by Bäck and Schwefel.[11] In the following, the main mechanisms of the multimembered evolution strategy, as shown in Fig. 5, are briefly discussed.

The strategy is based on the two main principles of natural evolution: mutation and selection. In an initialization step the first parent generation consisting of μ configurations (here $\mu = 15$) is generated stochastically. Every configuration is described by n object parameters and $n \times (n+1)/2$ strategy parameters. The design and with it the quality of a configuration are given by the object parameters. The strategy parameters represent the standard deviations and the linear correlation for the mutation of the object parameters; this means that the strategy parameters control the mutation of the object parameters. The strategy parameters are mutated as well. In doing so they adapt automatically during the optimization process. Schwefel[10] called this behavior self-adaptation.

The offspring generation is built up using global recombination; this means that every offspring get its parameters—object and strategy parameters—from all μ parents out of the parent generation. Which parameter is transmitted by which parent is selected by chance. In addition to the variation by recombination, a stochastic mutation takes place. This is necessary because recombination alone allows only a new mixture of parameters that already exist in the parent generation. Without additional mutation the optimal parameters already have to be present in the first parent generation if the optimal solution is to be found. Generally they are not. Therefore, a mutation is necessary because it guarantees new parameter values from generation to generation. By using recombination and mutation a total number of λ offspring (here $\lambda = 100$) is produced. Naturally, each of these λ offspring configurations must be rated. At this point of the algorithm there is a total population of $\mu + \lambda$ individuals. The question is how to select the new parent generation out of these $\mu + \lambda$ individuals. In the present study, the selection is done out of the λ offspring and the δ best parents (here $\delta = 2$). This kind of selection guarantees on the one hand that the best configuration is not lost and on the other hand that new configurations are taken over into the next parent generation in every selection step. Therefore the totality of parameters, in biological terms the gene pool, is renewed from generation to generation. Finally, the best μ configurations from the λ offspring and the δ possibly surviving parents are selected and constitute the new parent generation. Then the whole game of recombination, mutation, and selection starts again until a break criterion is reached.

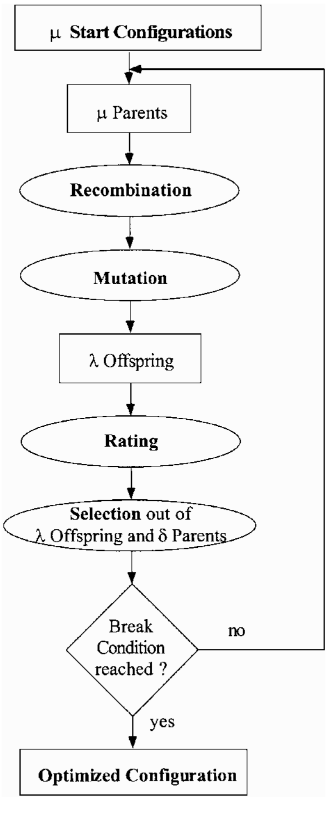

Fig. 5 Multimembered evolution strategy.

Optimization Results

Multidimensional optimizations were performed for the following design parameters: $(T/W_0)_T$, $(T/W_0)_R$, $M_{T\rightarrow R}$, z, $M_{z>1}$, and M_{sep}. The first five parameters are propulsion system parameters. The last one, the initialization Mach number for the stage separation, is actually a mission parameter. It is considered here in addition to the propulsion system parameters because it has a strong influence on the ramjet design. The reference value and the lower and upper parameter boundaries are given in Table 1 for each parameter. In addition to these boundaries two other constraints must be kept during optimization. The size ratio SR between first and second stage, given by

$$SR = \max(l_{\mathrm{SS}}/l_{\mathrm{FS}}, b_{\mathrm{SS}}/b_{\mathrm{FS}}) \tag{8}$$

must be lower than 50% to limit the structural load on the first stage and to allow the integration of the second stage. Furthermore, the volume utilization of the first stage, i.e., the ratio between the tank volume of the first stage and the total internal volume of the first stage, is limited to 50%.

In the sensitivity analysis, model inaccuracies were exclusively assumed in the physical computation models for the lower stage. Inaccuracies in the upper-stage models were not considered. In this paper the following six lower-stage models are involved in the sensitivity analysis: total drag, fuel mass flow, empty mass, propulsion mass, thrust turbojet mode, and thrust ramjet mode.

The assumed size of the inaccuracies Δ_i and of the weighting factors g_i were equal for each model and were arbitrarily set to 5% because this value is considered neither too optimistic nor too pessimistic.

Multidimensional optimizations were performed using the quality criteria defined earlier. The optimization results, including the optimized design parameters and the values of the different quality criteria, are presented in Table 2. The optimized values are boldfaced. The corresponding values of the reference propulsion design are added for comparison.

Table 1 Reference values and boundaries of the design parameters

Parameter	Reference	Minimum	Maximum
$(T/W_0)_T$	0.97	0.8	1.1
$(T/W_0)_R$	0.27	0.2	0.4
$M_{T\rightarrow R}$	3.0	3.0	3.4
z	——	1.0	3.0
$M_{z>1}$	——	6.0	7.5
M_{sep}	7.0	6.5	7.5

Table 2 Optimization results

	Optimized quality criterion					
Parameters/criteria	Ref.	GF	GF^+	S^+	GF^*	S^*
Design parameters						
$(T/W_0)_T$	0.97	0.85	0.89	0.97	0.93	1.04
$(T/W_0)_R$	0.27	0.25	0.25	0.35	0.25	0.33
$M_{T\to R}$	3.0	3.4	3.4	3.4	3.4	3.4
z	——	2.0	1.4	——	1.2	——
M_{st}	——	6.7	6.4	——	6.3	——
M_{sep}	7.0	7.3	7.5	7.5	7.5	7.5
Quality criteria						
GF	51.3	**47.1**	49.3	67.1	54.5	72.5
GF^+	68.4	64.6	**57.5**	70.4	59.5	75.9
S^+	17.1	17.5	8.2	**3.3**	5.0	3.4
GF^*	119.6	231.9	72.0	72.8	**64.8**	77.5
S^*	68.3	184.8	22.7	5.7	10.3	**5.0**

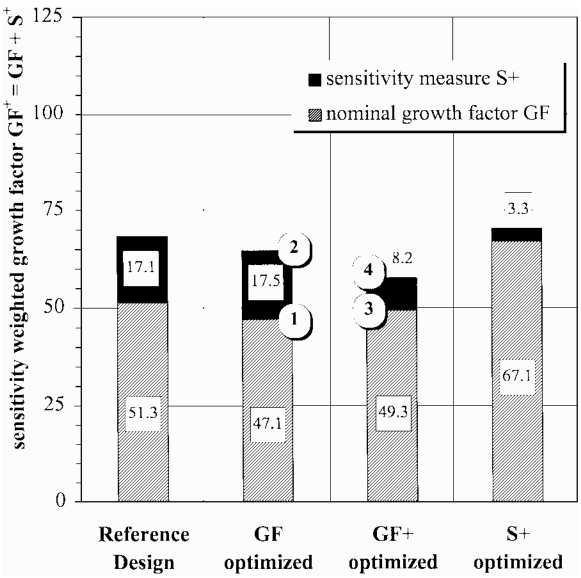

Fig. 6 Sensitivity-weighted growth factor GF^+ (weighted gradients approach).

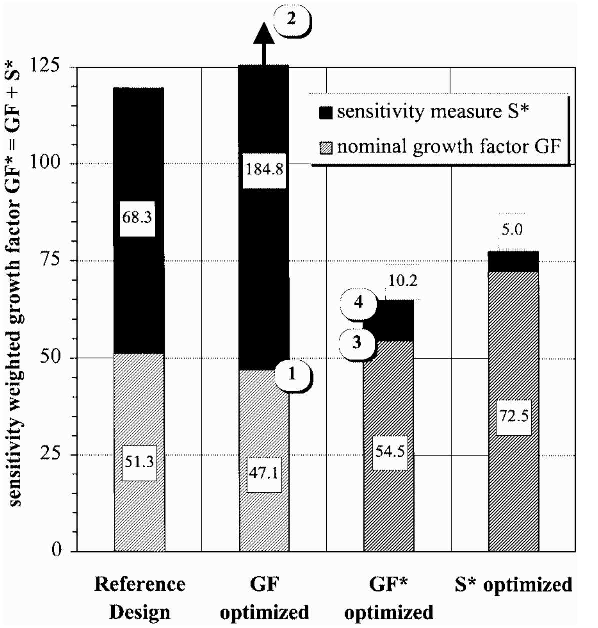

Fig. 7 Sensitivity-weighted growth factor GF^* (global sensitivity approach).

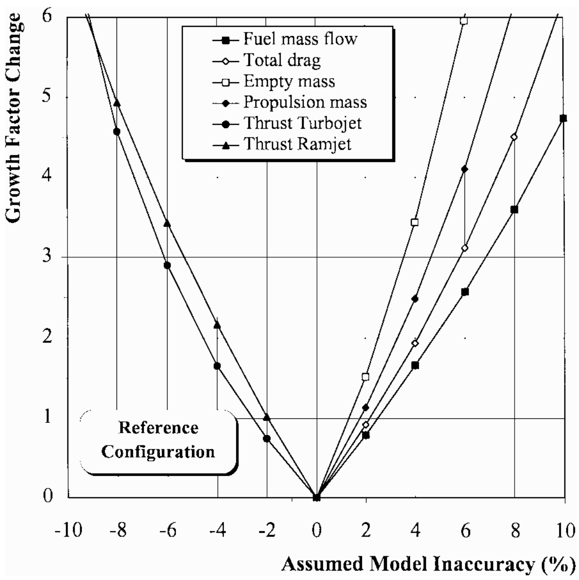

Fig. 8 Sensitivity of the reference propulsion design.

The optimization for a minimum nominal growth factor yields a smaller propulsion system in comparison to the reference propulsion system design. This is especially valid for the turbojet engines where the initial thrust-to-weight ratio is diminished by nearly 15% from $(T/W_0)_T = 0.97$ to 0.85, mainly resulting in a lower propulsion system mass. In contrast, the fuel mass is increased due to a lower acceleration capability, especially in the transonic regime. In addition, the initialization Mach number for the stage separation is increased from $M = 7.0$ to 7.3. The resulting lack of acceleration capability in the high-Mach-number regime is compensated by an overstoichiometric combustion with a stoichiometric ratio of $z = 2.0$ in the last flight phase before the stage separation ($M > 6.7$). Altogether, the nominal growth factor is improved from $GF = 51.3$ to 47.1.

The sensitivity measures S^+ according to the weighted gradients approach are nearly equal for both propulsion system designs ($S^+ = 17.1$ compared with 17.5), as shown in Fig. 6, but this changes dramatically if the sensitivity measures S^*, according to the global sensitivity approach, are considered (Fig. 7). In this case the sensitivity measure of the propulsion design that is optimized only for the best nominal growth factor is considerably higher than the sensitivity of the reference propulsion design ($S^* = 184.8$ compared with 68.3). This very high sensitivity is due to nonlinear effects that are included in the global sensitivity approach, and it shows impressively that critical situations can arise if only the nominal criterion is considered.

The reasons can be easily understood if the sensitivities to inaccuracies in the different models are compared for the reference propulsion design (Fig. 8) and the optimized nominal growth factor design (Fig. 9). The sensitivities with respect to inaccuracies in the turbojet thrust model and the drag model are higher for the optimized nominal growth factor design. In contrast, the sensitivities with respect to the mass models in general and the propulsion mass model in particular are reduced. Unfortunately, inaccuracies in the thrust and the drag models influence the space transportation system in the same way. They diminish the acceleration capability and thereby cause an increasing fuel mass. In the global sensitivity approach these inaccuracies work together, strengthen the negative effects in a nonlinear way, and lead in the end to the very strong increase in the sensitivity measure to $S^* = 184.8$.

A significant sensitivity reduction is reached if the sensitivity-weighted growth factor GF^+ according to the weighted gradients approach is used as optimization criterion. The total sensitivity measure S^+ decreases from $S^+ = 17.5$ to 8.2 (Fig. 6) due to lower sensitivities in each single model, but especially for the thrust, drag, and empty mass models, as shown in Fig. 10. Naturally, there is an increase in the nominal growth factor, but it is only a slight change from $GF = 47.1$ to 49.3. The effects described earlier are mainly due to an enlargement of the turbojet design. The initial thrust-to-weight ratio of the turbojet engines rises from $(T/W_0)_T = 0.85$

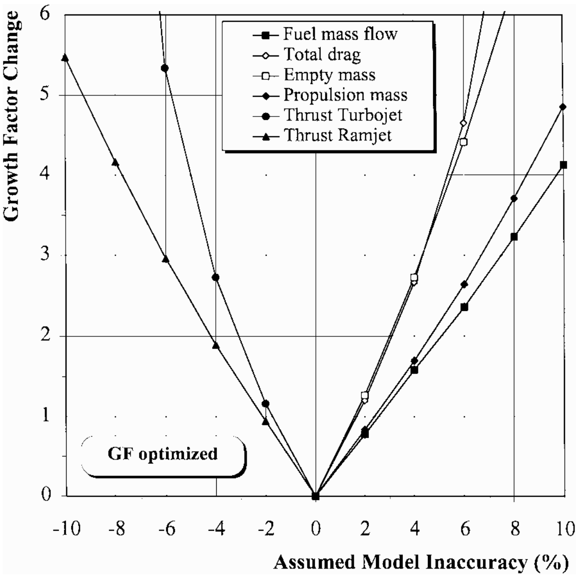

Fig. 9 Sensitivity of the propulsion design optimized for minimal nominal growth factor.

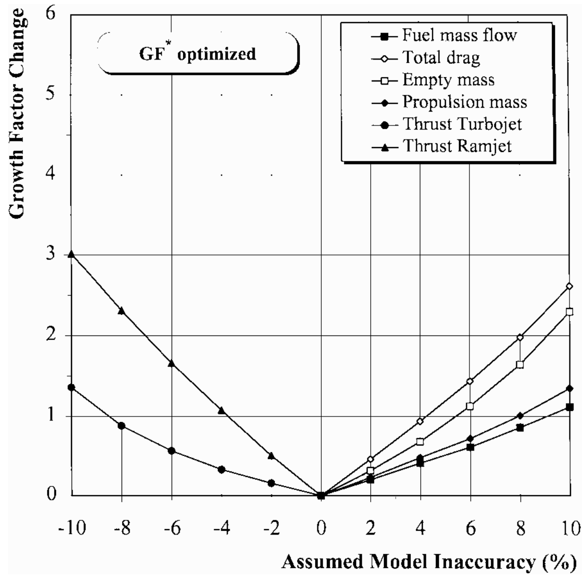

Fig. 11 Sensitivity of the propulsion design optimized for minimal sensitivity-weighted growth factor GF^* (global sensitivity approach).

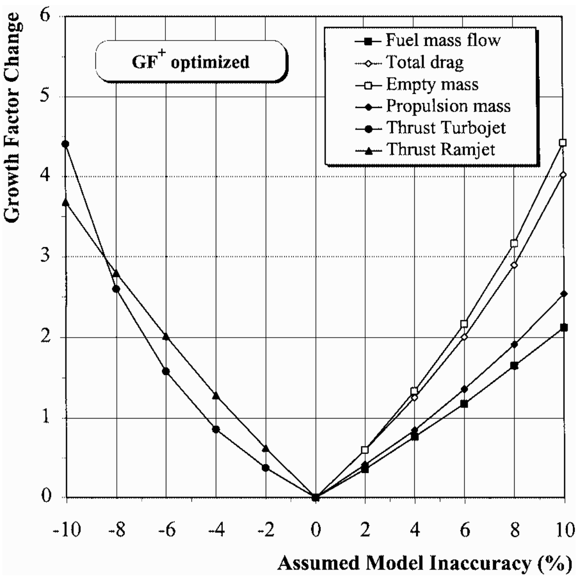

Fig. 10 Sensitivity of the propulsion design optimized for minimal sensitivity-weighted growth factor GF^+ (weighted gradients approach).

to 0.89, influencing the sensitivities to inaccuracies in the drag and the turbojet thrust models in particular.

As side effects, the stage separation initialization Mach number increases to its upper boundary value of $M = 7.5$, and the overstoichiometric ratio decreases, whereas the overstoichiometric combustion itself works over a longer period (Table 2). The design sensitivity can be reduced further if the sensitivity-weighted growth factor GF^* according to the global sensitivity approach is the decisive optimization criterion. As shown in Fig. 7, the sensitivity measure S^* decreases by more than an order of magnitude from $S^* = 184.8$ to 10.3. Of course, a price must be paid for this immense sensitivity reduction, i.e., an increase in the nominal growth factor. But the resulting increase in the nominal growth factor from $GF = 47.1$ to 54.5 turns out to be fairly moderate.

What are the reasons for the diminished sensitivity? In comparison with the optimal propulsion design, according to the weighted gradients approach, the initial thrust-to-weight ratio of the turbojet is increased from $(T/W_0)_T = 0.89$ to 0.93, further reducing the sensitivity to inaccuracies in the turbojet thrust model and in the drag model, as shown in Fig. 11, by improving the acceleration capability mainly in the transonic regime. The other tendencies described earlier are confirmed as well. The stage separation initialization Mach number is set to its upper limit, and the overstoichiometric ratio decreases, whereas the overstoichiometric part of the trajectory is extended.

Note that, in analogy to the one-dimensional example, shown in Fig. 1, the characteristic points 1–4 are added to the bar charts in Figs. 6 and 7, indicating that the basic idea of the sensitivity-based design optimization that was derived from a one-dimensional example also proved good in a multidimensional case.

A further sensitivity reduction is possible if the sensitivity measures S^+ or S^* are used as optimization criteria. In this case the reduced sensitivity ($S^+ = 3.3$ and $S^* = 5.0$) implies a considerable increase in the nominal growth factor to $GF = 67.1$ and 72.5, respectively, and it must be questioned whether these nominal performance losses are acceptable.

It should be noted that the remaining sensitivity in the case of optimized sensitivity measures mainly refers to the sensitivities with respect to inaccuracies in the ramjet thrust model and in the drag model. The sensitivities with respect to the other models are almost completely reduced. An interesting fact is that the propulsion design optimized exclusively for minimal sensitivity makes no use of overstoichiometric combustion (Table 2). If the acceleration capability is not sufficient to reach the desired separation Mach number in the case of model inaccuracies, these configurations start the stage separation process earlier and transfer a part of the first-stage mission to the second stage. In this case the fuel mass of the second stage is increased and the payload decreased.

Summary

In this paper a new method for the performance rating of a space transportation system was presented. It combines a commonly used quality criterion, the so-called growth factor, with a penalty function that penalizes sensitivities with respect to inaccuracies in the physical computation models used in the mission simulation. Two approaches for this penalty function were introduced. The first one, weighted gradients, is based on a linear composition of sensitivities to inaccuracies in one submodel; the second one, global sensitivity, refers to inaccuracies in all submodels at the same time.

The method was successfully applied to a TSTO space transportation system with horizontal takeoff and landing, an air-breathing first stage, and a rocket-propelled second stage. An optimization of the propulsion system of the first stage was performed with and without considering design sensitivities, using an evolution strategy as the optimization method.

It could be shown that remarkable sensitivity reductions (more than 95%) are achievable by using a sensitivity-weighted growth factor as an optimization criterion. The resulting losses in the nominal performance turned out to be relatively small.

The comparison of the two approaches shows that the weighted gradient approach is especially suitable to identify the models that make the biggest contribution to the total sensitivity. This means that this approach yields helpful hints about which models should be treated very carefully and at least should be improved. For the evaluation of the total system, the global sensitivity approach should be used because the nonlinear interactions of the inaccuracies in several models, which are important sensitivity drivers, are included.

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J. A. Martin
Associate Editor